Examination of Current-Induced Magnetic Field in the Slab Geometry: Possible Origin of Spin Hall Effect

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We estimate the strength of current-induced magnetic field (CIMF) in the two-dimensional slab geometry for Spin Hall Effect (SHE) observed recently by Kato *et al.* and Wunderlich *et al.* and show that if the factor gm^*/m, where g is the Lande factor and m^* and m are effective and pure masses, respectively, is equal to the numerical value at the surface of the semiconductor, then the CIMF can describe the SHE.

I.   INTRODUCTION

Recent observation of electron Spin Hall Effect (SHE) by Kato *et al.* [1] and hole SHE by Wunderlich *et al.* [2] has attracted much attention from condensed-matter physicists, since spin polarization on nonmagnetic semiconductor thin-film edges has been induced by longitudinal electric current. Spins [1] are polarized along the direction perpendicular to the current. However, Kato *et al.* claimed that the current-induced spin-polarization should be below the experimental capability to detect it.

II.   THEORY AND MATHEMATICAL TREATMENT

For a theoretical explanation of the SHE, theories based on the spin-orbit (SO) interaction have been widely developed. Although the SO interaction is relativistic [3] of second order on ratio of electron velocity to light velocity, it is supposed [4] that a band splitting induced by the SO interaction is of the same scale as the energy gap between the conduction and valence bands. Two kinds of microscopic scenarios employing the SO interaction have been investigated: extrinsic [5] as a result of asymmetric scattering for up and down spins, and intrinsic [6] connected only with the band structure of semiconductors. These approaches have been extensively discussed [7] in the literature. Nomura *et al.* [8] combined the theoretical calculations for the intrinsic effect with the experimental data. However, among these complicated treatments, it seems that the role of the Zeeman splitting of the electronic energies by

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the current-induced magnetic field (CIMF) for the SHE has so far not been sufficiently investigated. Kato *et al.* considered that the CIMF strength is not large enough to explain the observed SHE. In this paper, we show that, by taking into account some material parameters, the CIMF can be responsible for the SHE and the spin polarization of the electron gas in semiconductors. Namely, we will show that the estimated CIMF on edges of slab geometry of samples and the numerical value of the factor gm^*/m on the surface of the semiconductor can provide a sufficient value required for SHE. More explicitly, we show that the component of the CIMF perpendicular to the plane of a slab around the edge is divergent as the logarithm of the ratio of the width to the thickness of the slab. It is zero at the middle of the width. Therefore, in the case of infinitesimal thickness, the spins might be polarized perpendicular to the semiconductor mainly around the edges. This structure of the magnetic field might provide the asymmetric Hall Effect, when both edges of the sample are charged with same-sign charges (we have called the effect asymmetric because in the conventional symmetric Hall Effect the two edges are charged with opposite-sign charges). Additionally, we will repeat the calculation by Kato *et al.* to estimate the strength of CIMF and confirm that it equals ours. At the condition $gm^*/m \rightarrow 2$, *i.e.*, when the factor gm^*/m tends to have a vacuum value on the surface of the semiconductor (we can assume that the surface-monolayer non-magnetic atoms are to be in vacuum and put for their electrons $m^* \approx m$ and $g = 2$ [9]), the numerical value of CIMF strength will be enough to create experimental SHE.

The absolute value of magnetic field $\vec{H}$ for the infinite length cylindrical-geometry metallic sample, as a solution of the Maxwell equation, $\vec{\nabla} \times H = (4\pi/c)\vec{j}$, written in the integral form, $\oint \vec{H} d\vec{l} = (4\pi/c) \int \vec{j} d\vec{f}$, has the form $H = 2J(r)/(cr)$, if the density of the current $j(r)$ is constant at a fixed radial distance r from the longitudinal axis of the conductor. Here, magnetic field $\vec{H}$ is along the closed curve $\vec{l}$ with radius r, taken around the conductor on a transverse section to it, and $J(r)$ is the current flowing through the area πr^2.

The experimental film slab can be modeled as a finite number of thin cylindrical conductors connected with each other. At a lateral coordinate x of a two-dimensional conductor of width L, when a constant current density J is assumed to flow in the z-direction, the magnetic field along a direction (y-direction) perpendicular to the surface is calculated as

$$H = \frac{2}{c} \int_0^L \frac{d\,X j(X)}{(X-x)} , \tag{1}$$

and then $H = (2j/c)Ln(|x-L|/|x|)$ if j is constant. Magnetic field H diverges at the edge, as shown in Figure 1, and becomes zero at the center of the width of the slab. On the other hand, for a slab-geometry thin-film conductor with thickness d, the numerically simulated dependence of H as a function of x shows logarithmic divergence of H when we decrease d at fixed L.
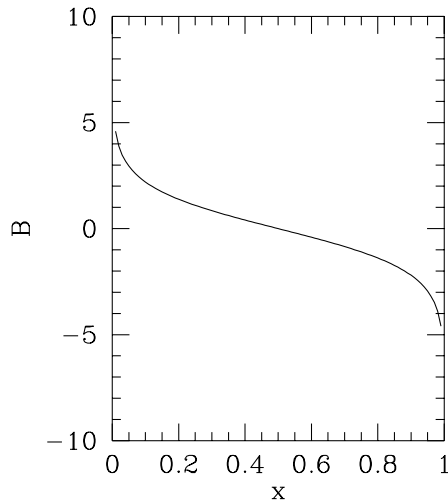
FIG. 1: Plot of the dependence of the component of magnetic induction $10^{-1}B/(\mu_0\mu)$ (in A/m units) perpendicular to width L of the sample as a function of observable point coordinate x. Numerical values of magnetic-permeability constants μ_0 and μ are described in the text.

To examine whether, indeed, this CIMF is responsible for the SHE or not, we evaluate the strength of magnetic field required for the spin polarization of the carriers indicated in the experiment and compare it with that for CIMF.

III.   THE GAS OF ELECTRONS IN THE EXPERIMENT OF KATO *ET AL.* [1]

For comparison, we consider the case of the strained semiconductor, because there is information in this on the density of polarized electrons. The gas of electrons in the heterostructure n-type $In_{0.07}Ga_{0.93}As$ is provided by doped Si atoms with density 3×10^{16} cm^{-3}; therefore, the density of electrons is $\rho = 3 \times 10^{16}$ cm^{-3}. The thickness and width of the sample are $d = 500$ nm and $L = 33$ μm, respectively. The SHE measurement was carried out at $10\,K < T < 60\,K$. The density of polarized electrons had the systematic error in the interval $+48 \div (-38)$ percent; therefore, we use the ratio of the density of polarized electrons to the full density of electrons: 10^{-4} [10]. The data for the Lande factor in the bulk of a semiconductor [10], effective mass [11] and mobility [12] of electron carriers are the following: $g = 0.64$, $m^* = 0.068m$, and $\mu_e \approx 5400$ cm^2/($V \cdot s$) (for mobility, we used the value for n-type Si-doped $GaAs$ with an electron density of 10^{16} cm^{-3}). The information for experimental effective mass can be compared with Ref. [13] for $In_{0.08}Ga_{0.92}As/GaAs$ and Refs. [14] for other semiconductor materials.

We find the following numerical ratios: $\lambda_B/d \approx 2 \cdot 10^{-2}$ and $\lambda_B/L = 1.351 \cdot 10^{-4}$, where $\lambda_B = \hbar/p_F$ is the de Broglie wavelength with Fermi momentum $p_F = (3\pi^2\rho)^{1/3}\hbar$. The calculation of the numerical value of the Fermi

energy $\mathcal{E}_{\mathcal{F}}/k_B = D\rho^{2/3}/k_B$ [15], where $D = (3\pi^2)^{2/3}\hbar^2/(2m^*)$ and k_B is the Boltzmann constant, expressed in Kelvin temperature units (K) and measured from the bottom of the conduction band, gives $\mathcal{E}_{\mathcal{F}}/k_B = 60.13\,K$. These values indicate that the electron gas can be treated as three-dimensional at the measured temperatures and Fermi-degenerate. The particles on the Fermi surface are quasi-classical.

To evaluate the strength of magnetic field required for the spin polarization of electrons in the semiconductor, we use the simple scheme [16] of explanation of the effect. There is a Zeeman splitting $\mathcal{E}^{\uparrow,\downarrow} = \mathcal{E}_{\mathcal{F}} \pm g\mu_B H$ of two subbands of electrons with spin-up $s_z = +1/2 = \uparrow$ and spin-down $s_z = -1/2 = \downarrow$ directions of spins in the external magnetic field. We note that in this definition of directions of spins, the spin-up component is parallel to the magnetic-field vector [17]. Each energy in $\mathcal{E}^{\uparrow,\downarrow}$ is measured from the bottom of its own subband of conductance. In the absence of a magnetic field, the numbers of spin-up and spin-down electrons are equal; therefore, there is no spin polarization. When a magnetic field is applied, the subband of spin-up electrons shifts down, while the subband of spin-down electrons goes up. As the Fermi energy is the same for both spin components of the gas, the electrons with spin-down spins, whose energy is above the Fermi energy, undergo spin-flipping and occupy the opened free levels in the subband of spin-up electrons, below the Fermi energy. The gas has spin-up polarization along the external magnetic field. In the geometry of the Kato *et al.* experiment, the direction of polarized spins on the edges coincides with the direction of CIMF, which can be a qualitative indication that CIMF is responsible for SHE.

From the above explanation of spin polarization, assuming that $\mathcal{E}^{\uparrow,\downarrow} = D(\rho^{\uparrow,\downarrow})^{2/3}$, we obtain the strength of magnetic field:

$$H_e = \frac{D}{2g\mu_B}((\rho^{\uparrow})^{2/3} - (\rho^{\downarrow})^{2/3}) \qquad (2)$$

for polarization of $\rho^{\uparrow} - \rho^{\downarrow}$ density of electrons with spin-up direction of spins. Using the expression $\mu_B = |e|\hbar/(2mc)$ for the Bohr magneton, $\rho^{\uparrow} - \rho^{\downarrow} = 10^{-4}\rho$, $\rho^{\uparrow} + \rho^{\downarrow} = \rho$, and substituting the numerical values for the quantities gm^*/m and elementary flux quantum of magnetic field $\phi_0 = \pi\hbar c/|e|$ in Eq. (2), we find $H_e = 56.771 \cdot 10^{-4}$ T. This is an estimate of the strength of magnetic field required for observation of SHE in the paper of Kato *et al.*

For the calculation of the strength of CIMF, we use the expression

$$H \approx \frac{I}{2\pi L} Ln\left(\frac{L}{d}\right). \qquad (3)$$

Here, H is expressed in SI units (for that, we replaced the coefficient $4\pi/c \to 1$) and I is the electric current flowing along the longitudinal direction of the sample. In the experiment of Kato *et al.*, one gives the electric field $E = 25$ mV/μm instead of current I. Employing the relations $j = \sigma E$ between the density of current j and E, and $\sigma = \rho|e|\mu_e$

between conductivity σ and mobility of electrons μ_e, we find the numerical value for j. Then, substituting the data for L and d for the determination of I through j in Eq. (3), one derives the numerical value $33.526\,A/m$ for H. The magnetic induction B_e and H are connected with each other *via* the expression $B_e = \mu_0\mu H$, where $\mu_0 = 4\pi \cdot 10^{-7}\,H/m$ is the magnetic permeability of vacuum and $\mu = 1 + \chi_P$ with Pauli magnetic susceptibility χ_P. For the three-dimensional electron gas [17] $\chi_P = \mu_B^2 p_F m^*/(\pi^2\hbar^3)$ and, using the data for density ρ and m^*, we obtain $\chi_P = 18.667 \cdot 10^{-10}$. Therefore, the numerical value for strength of CIMF is $B_e = 4.213 \cdot 10^{-5}$ T. The ratio between B_e and H_e is $B_e/H_e = 7.421 \cdot 10^{-3}$, which gives the estimate 10^{-6} of Kato *et al.* for the polarization degree expected from B_e. However, on the surface of the semiconductor $gm^*/m \to 2$, *i.e.*, should be close to the vacuum value, and then the real quantity is $H_e = 1.235 \cdot 10^{-4}$ T; hence, H_e has the same order of magnitude as B_e.

IV.   THE GAS OF HOLES IN THE EXPERIMENT OF WUNDERLICH *ET AL.* [2]

The experiment was performed on $(Al, Ga)As$ film doped with acceptor Be. The size parameters of the sample are $d = 1$ nm [11] and $L = 1.5\ \mu$m. Other quantities describing the experiment are the following: regime for temperature $T = 4.2$ K, effective mass $m^* = 0.27m$, current of holes $I_p = 100\ \mu$A, two-dimensional density of holes $n = 2 \cdot 10^{12}$ cm^{-2}, $g = 0.5$ (this value has been supposed for g, due to its absence in the literature for the investigated or related materials). As for the Kato *et al.* gas of electrons, we assume that the ratio of density of polarized holes to full density of holes is 10^{-4}. The three-dimensional density of holes will be $\rho_h = n/d$, and we obtain $\rho_h = 2 \cdot 10^{19}\,cm^{-3}$.

We have $\lambda_B/d \approx 1.19$ and $\lambda_B/L = 0.079 \cdot 10^{-2}$ for this value of ρ_h; therefore, the gas of holes is two-dimensional. The numerical value of the Fermi energy $\mathcal{E}_\mathcal{F}{}^0/k_B = An/k_B$, where $A = \pi\hbar^2/m^*$, of this gas of holes, measured now from the top of the valence band, yields $\mathcal{E}_\mathcal{F}{}^0/k_B = 2.059 \cdot 10^2\,K$, which means that at experimental temperature the gas is Fermi-degenerate.

The Zeeman splitting for the two-dimensional holes is described by the expression $\mathcal{E}^{\uparrow,\downarrow} = \mathcal{E}_\mathcal{F}{}^0 \mp g\mu_B H$. Hence, one polarizes the spin-down $s_z = -1/2 = \downarrow$ holes (again, according to the definition, the spin $s_z = +1/2 = \uparrow$ is parallel to the magnetic-field vector). Assuming $\mathcal{E}^{\uparrow,\downarrow} = An^{\uparrow,\downarrow}$, we obtain for the strength of magnetic field the expression

$$H_h = \frac{A}{2g\mu_B}(n^\downarrow - n^\uparrow) \tag{4}$$

for polarization of $n^\downarrow - n^\uparrow$ density of holes with spin-down direction of spins. Substituting $n^\downarrow - n^\uparrow = 10^{-4}n$ and other quantities, as has been performed above for a three-dimensional electron gas, in Eq. (4), one obtains $H_h = 148.15 \cdot 10^{-4}$ T. On the other hand, taking into account that for the present two-dimensional gas of holes the Pauli susceptibility $\chi_{P,h} = \mu_B^2 m^*/(\pi\hbar^2 d)$ is $\chi_{P,h} = 6.056 \cdot 10^{-8}$, the application of Eq. (3) with parameters $d = 1$ nm, $L = 1.5\ \mu$m and

$I_p = 100$ μA yields $B_h \approx 10^{-4}$ T; therefore, $B_h/H_h = 0.00676$. On the assumption $gm^*/m \approx 2$ on the surface of the semiconductor, we derive $H_h = 10^{-3}$ T. B_h should be increased slightly, due to size quantization in the thickness direction of the sample (for the experimental temperature, one performs the condition $T \ll \hbar^2/(m^*d^2k_B)$ and the gas is in the ground state of the approximately descriptive one-dimensional infinite rectangular well, the wave function of which has the radius localization d/π). Hence, H_h and B_h have the same order of magnitude.

Finally, we need to make the following remark. In the calculation of $B_{e,h}$, it has been supposed that the numerical factor inside of the logarithmic function in Eq. (3) is unity. However, one can show that for a thin slab it tends to 4. Hence, the numerical value of $B_{e,h}$ becomes closer to that of $H_{e,h}$. On comparing the results obtained for both experiments, one can conclude that a possible real estimate for the strength of magnetic field for the observation of SHE is $H_{e,h} \approx B_{e,h} \sim 1$ mT. At last, the structure of CIMF allows us to explain the spatial dependence of spin lifetime across the sample of Kato *et al.*, as long as it is proportional to the modulus of magnetic field, and to predict the Asymmetric Hall Effect. In this effect, the two edges of the sample are charged with same-sign charges.

V.   SUMMARY

We have investigated an estimate for the strength of CIMF and shown that it can be close to that required for the observation of SHE in the experiments of Kato *et al.* [1] and Wunderlich *et al.* [2]. The reason is that the parameter gm^*/m on the surface of the semiconductor could have the numerical value for vacuum. Two qualitative results obtained might support the fact that CIMF is responsible for SHE. First, the calculated component of CIMF, being perpendicular to the main surface of the slab, shows logarithmic divergence of the ratio of width to thickness of the sample with opposite signs on the edges, and, second, the direction of polarized spins in the experiment of Kato *et al.* [1] is along the CIMF, which is expected for SHE in this magnetic field. From the structure of CIMF, one could also progress to the prediction of the Asymmetric Hall Effect, when both edges of the sample could be charged with same-sign charges.

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